UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 2123
COURSE	: STATISTICS
SEMESTER/SESSION	: 1-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **11** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 11 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS (100 MARKS)

QUESTION 1

a) A survey will be given to 100 students randomly selected from the freshmen class at Lincoln High School. What is the population and sample? (2 marks)

b) A high school teacher report that her class's average score on a final exam was 72. Is this figure a parameter or a statistic? Explain. (2 marks)

c) Kasyaf is carrying out a survey. He would like to find out how many kilometers people drive per day. He divides the population into a two way table, and calculates the proportion of each category required for his sample. Which sampling technique is Kasyaf using? (1 mark)

QUESTION 2

Each day, x, the number of diners in a restaurant was recorded and the following grouped frequency table was obtained.

x	16-20	21-25	26-30	31-35	36-40
No of days	67	74	38	39	42

a) Calculate the mean for the above distribution. (3 marks)

b) Calculate the mode for the above distribution. (4 marks)

c) Calculate the standard deviation for the above distribution. (4 marks)

QUESTION 3

Let A and B be events such that $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{4}$ and $P(A\,or\,B)=\frac{11}{20}$. Find each of these probabilities:

a) $P(A\,and\,B)$ (3 marks)

b) $P(A|B)$ (2 marks)

c) Are the events A and B are independent? (2 marks)

d) Are the events A and B are mutually exclusive? (2 marks)

QUESTION 4

Three data entry specialist enter requisitions into a computer. Specialist 1 processes 30 percent of the requisitions, specialist 2 processes 45 percent and specialist 3 processes 25 percent. The proportions of incorrectly entered requisitions by data entry specialist 1, 2 and 3 are 0.03, 0.05 and 0.02 respectively. Suppose that a random requisition is found to have been incorrectly entered. Find the probability that:

a) A random requisition is found to have been incorrectly entered? (3 marks)

b) The data entry specialist 1, given a random requisition is found to have been incorrectly entered. (2 marks)

QUESTION 5

The discrete random variable X has a probability distribution function given by

x	1	2	3	4	5
$P(X=x)$	$0.2k$	$0.5k$	$0.3k$	$0.4k$	$0.6k$

Find:

a) The value of k. (3 marks)

b) $P(X \leq 4)$ (2 marks)

c) $E(X)$ (3 marks)

QUESTION 6

The number of accidents in a randomly chosen week at a factory can be modeled by a Poisson distribution with mean 0.7. Find the probability that:

a) No accident in a randomly chosen week. (2 marks)

b) More than two accidents in a randomly chosen week. (4 marks)

QUESTION 7

The mass of a small loaf of bread is normally distributed with mean 500g and standard deviation 20g. Find the probability that a randomly chosen loaf has a mass:

a) At most 510g. (3 marks)

b) At least 515g. (3 marks)

QUESTION 8

Independently for each year, the number of road accidents per year at a certain black spot may be regarded as having a Poisson distribution with 2.3 as mean. Find the probability:

a) More than two accidents will occur in a given year using the Poisson distribution. (3 marks)

b) At least 30 accidents in a given ten-year period using the Normal approximate to the Poisson distribution. (5 marks)

QUESTION 9

A random sample of 120 library books is taken as they are borrowed. They are classified as fiction or non- fiction and hardback or paperback. 88 books are found to be fiction and 74 are paperback. Find a 90% confidence interval for:

a) The proportion of books borrowed that are fiction. (5 marks)

b) The proportion of books borrowed that are paperback. (5 marks)

QUESTION 10

A study included 123 children who were wearing seat belts when injured in motor vehicle crashes. The amounts of time spent in an intensive care unit have a mean of 0.83 day and a standard deviation of 0.16 day.

a) Using 0.01 significance level, test the claim that the seat belt sample comes from a population with a mean of less than 1.39 days, which is the mean for the population who were not wearing seat belts when injured in motor vehicle crashes. Use 5 step of hypothesis testing.

(13 marks)

b) Develop a 95% confidence interval for the mean for the population who were not wearing seat belts when injured in motor vehicle crashes. (5 marks)

QUESTION 11

The yield of a particular crop on a farm is thought to depend principally on the amount of rainfall in the growing season. The value of the yield y, in tons per acre, and the rainfall x, in centimeters, for seven successive years are given in the table below.

x	12.3	13.7	14.5	11.2	13.2	14.1	12.0
y	6.25	8.02	8.42	5.27	7.21	8.71	5.68

a) Find the Pearson's correlation between x and y. Explain the value. (6 marks)

b) Find the equation of the least squares line fit to these data. (6 marks)

c) Given that the rainfall in the growing season of a subsequent year was 14 cm, estimate the yield in that year. (2 marks)

----------End of questions----------

FORMULA

Relative frequency $= \dfrac{f}{\sum f}$	Midpoint, $x = \dfrac{\text{Lower limit} + \text{Upper limit}}{2}$
$\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$	$\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
$Median, \tilde{x} = L_m + \left(\dfrac{\frac{\sum f}{2} - \sum f_{m-1}}{f_m} \right) \times C_m$ L_m = Lower bound of median class f_m = Frequency ofmedian class $\sum f_{m-1}$ = Cumulative frequency before median class C_m = Size of median class $\sum f$ = Number of observation /total frequency	$s^2 = \dfrac{1}{\sum f - 1}\left(\sum f_i x_i^2 - \dfrac{\left(\sum f_i x_i\right)^2}{\sum f} \right)$ $s = \sqrt{s^2}$
$Mode = L_{mo} + \left(\dfrac{d_1}{d_1 + d_2} \right) \times C_{mo}$ L_{mo} = Lower bound of mode class C_{mo} = Size of mode class d_1 = Difference between modal class frequency and the previous class frequency d_2 = Difference between modal class frequency and the next class frequency	

Probability

1. $P(A)=\dfrac{n(A)}{n(S)}$

2. a) $P(A\cap B)=P(B)\cdot P(A\mid B)$

   b) $P(A\cap B)=P(A)\cdot P(B)$ if and only if A and B are independent events.

   c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

   d) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$

3. $P(B_i\mid A)=\dfrac{P(B_i)\cdot P(A\mid B)}{P(A)}$

4. a) $P(X=x)=\binom{n}{x}p^x q^{n-x} \quad x=0,1,..,n$ b) $P(X=x)=\dfrac{\lambda^x e^{-\lambda}}{x!} \quad x=0,1,..$

5. a) $Z=\dfrac{X-np}{\sqrt{npq}}$ b) $Z=\dfrac{X-\lambda}{\sqrt{\lambda}}$

6. $E(X)=\sum_{-\infty}^{\infty}x\cdot f(x)=\int_{-\infty}^{\infty}x\cdot f(x)\,dx$

7. $E(X^2)=\sum_{-\infty}^{\infty}x^2\cdot f(x)=\int_{-\infty}^{\infty}x^2\cdot f(x)\,dx$

8. $Var(X)=E(X^2)-\left[E(X)\right]^2$

Regression and Correlation

1. $$r=\frac{(\Sigma xy)-\dfrac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left[(\Sigma x^2)-\dfrac{(\Sigma x)^2}{n}\right]\left[(\Sigma y^2)-\dfrac{(\Sigma y)^2}{n}\right]}}$$

2. $$b=\frac{(\Sigma xy)-\left(\dfrac{(\Sigma x)(\Sigma y)}{n}\right)}{\left((\Sigma x^2)-\dfrac{(\Sigma x)^2}{n}\right)} \qquad a=\frac{(\Sigma y)}{n}-b\frac{(\Sigma x)}{n}$$

Confidence Interval and Hypothesis Testing

Estimator	Confidence Interval	Test Statistics
$\mu_1 - \mu_2$	$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$Z_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $\nu = n_1 + n_2 - 2$	$T_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
	$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $\nu = n_1 + n_2 - 2$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$T_0 = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Confidence Interval and Hypothesis Testing

Estimator	Confidence Interval	Test Statistics
μ	$(\bar{x}) - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < (\bar{x}) + z_{\alpha/2}\frac{s}{\sqrt{n}}$	$Z_o = \frac{\bar{X} - \mu_o}{s/\sqrt{n}} \sim N(0,1)$
	$(\bar{x}) - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} < \mu < (\bar{x}) + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$	$T_o = \frac{\bar{X} - \mu_o}{s/\sqrt{n}} \sim t_{n-1}$

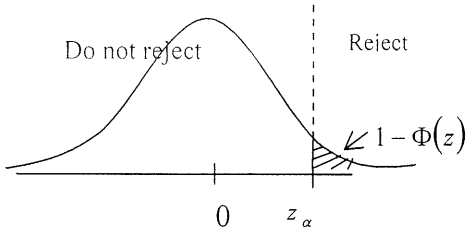
Estimator	Confidence Interval	Sample size
$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$n = \frac{z_{\alpha/2}^2 \cdot \hat{p}(1-\hat{p})}{E^2}$

APPENDIX I

Table I Standard Normal Distribution

$$1-\Phi(z)=P(Z>z)=\frac{1}{\sqrt{2\pi}}\int_{z}^{\infty}e^{-z^2/2}dz$$

$$z=\frac{x-\mu}{\sigma}$$



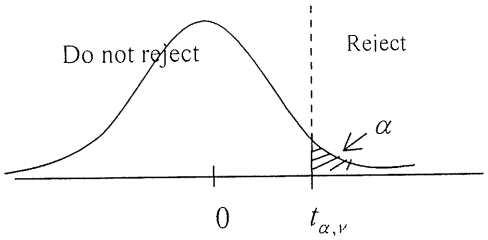
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
3.7	.000108	.000104	.000100	.000096	.000092	.000088	.000085	.000082	.000078	.000075
3.8	.000072	.000069	.000067	.000064	.000062	.000059	.000057	.000054	.000052	.000050
3.9	.000048	.000046	.000044	.000042	.000041	.000039	.000037	.000036	.000034	.000033
4.0	.000032									

5.0 → 0.0000002867 5.5 → 0.0000000190 6.0 → 0.0000000010

APPENDIX II

Table II Student's t Distribution

The table gives the value of $t_{\alpha,\nu}$ - the $100\,\alpha$ percentage point of the t distribution for degrees of freedom.



ν \ α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291