UNIVERSITY COLLEGE
TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BGE 1143
COURSE	:	DISCRETE MATHEMATICS
SEMESTER/SESSION	:	1 – 2024/2025
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **6** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches whenever required.
4. If in doubt, raise your hand and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

a) State which of the following are statement.

i. Switch off the lamp.

ii. Ahmad is a nice person.

iii. $x+y=12$

iv. It is hot today. (2 marks)

b) Determine the truth value of each compound statements.

i. $1>0$ and today is Friday. (1 mark)

ii. $1+1=3$ or $-5<1$. (1 mark)

c) Determine whether $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are logically equivalent or not. (6 marks)

d) Given that $X=\{x \in \mathbb{Z} \mid -2<x \leq 5\}$, $Y=\{y \in \mathbb{Z}^{+} \mid (y-2)(y-3)(y+1)=0\}$ and $Z=\{z \in \mathbb{R} \mid z^{2}=4\}$.

i. List all the elements in each of these sets. (3 marks)

ii. Find $X \cup Y \cup Z$, $X \cap Y$, $X-Y$ and $Z-(X \cap Y)$. (4 marks)

QUESTION 2

Prove by the mathematical induction that:

a) $\frac{1}{1 \cdot 2}+\frac{1}{2 \cdot 3}+\cdots+\frac{1}{(n-1) \cdot n}=1-\frac{1}{n}$ for $n \geq 2$. (8 marks)

b) $2+2^{2}+2^{3}+\cdots+2^{n}=2\left(2^{n}-1\right)$ for $n \geq 1$. (8 marks)

QUESTION 3

a) Let $p = 231$ and $q = 315$. Find the greatest common divisor (gcd) and the least common divisor (lcm) for both integers p and q by using prime factorization.

(4 marks)

b) Find the binary and hexadecimal of 501_{10}. (4 marks)

c) Given that $A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 1 & 7 \end{bmatrix}$. If possible, compute AC and $3A + BC$. (4 marks)

d) Given the Boolean matrices,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

or and boolean

Compute $A \vee B$, $A \wedge B$ and $A \odot B$. (3 marks)

QUESTION 4

a) Let set $A = \{a, b, c\}$, $B = \{1, 2\}$ and define relations $R = \{(a,b) \mid a \neq b\}$ and $S = \{(a,b) \mid a + b \leq 3\}$ from A to B. State explicitly which ordered pairs are in

i. $A \times B$ (1 mark)

ii. R (2 marks)

iii. S (2 marks)

b) Given $A = \{a, b, c, d, e\}$ and a binary relation R on A as follow,

$$R = \{(a,a), (a,c), (b,b), (b,d), (c,a), (c,c), (d,d), (d,e), (e,e)\}.$$

i. Draw a digraph for the relation R. (2 marks)

ii. Determine whether or not the binary relations R defined on the sets A are reflexive, symmetric or transitive. Explain why or why not? (3 marks)

c) Let $A = \{1,2,3\}$. List the ordered pairs in relation on A, which is:

i. R_1 is not reflexive, not symmetric and not transitive. (2 marks)

ii. R_2 is reflexive, but neither symmetric nor transitive. (2 marks)

iii. R_3 is reflexive and transitive, but not symmetric. (2 marks)

iv. R_4 is symmetric and transitive, but not reflexive. (2 marks)

QUESTION 5

a) Suppose a graph has five vertices of degrees 1, 1, 4, 4 and 6.

i. By using handshaking theorem, determine how many edges the graph has. (2 marks)

ii. Draw a possible graph whose degree sequence is the one specified. (2 marks)

b) Draw K_7 and $K_{3,4}$. (3 marks)

c) Figure 1 shows a graph.

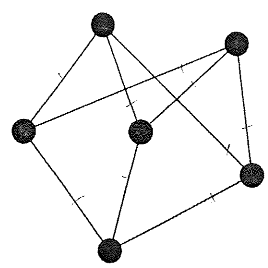


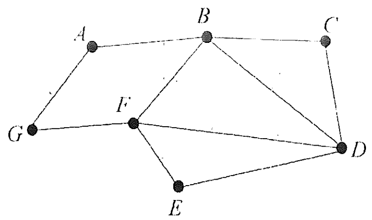
Figure 1

i. State the number of edges and vertices of the graph. (2 marks)

ii. Determine whether or not the graph is bipartite. Give the bipartition sets or explain why the graph is not bipartite. (2 marks)

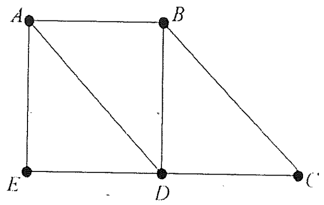
d) Determine whether each the following graph has an Euler path or Euler circuit. Construct such path or circuit if exist. If it does not exist, justify your reason.

i.



(3 marks)

ii.



(3 marks)

QUESTION 6

a) Show that in a Boolean algebra, for any A, B and C.

i. $A\overline{B}C + A\overline{B}\,\overline{C} = A\overline{B}$ (2 marks)

ii. $ABC + \overline{A} + A\overline{B}C = \overline{A} + C$ (4 marks)

b) Given a Boolean circuit as shown in Figure 2.

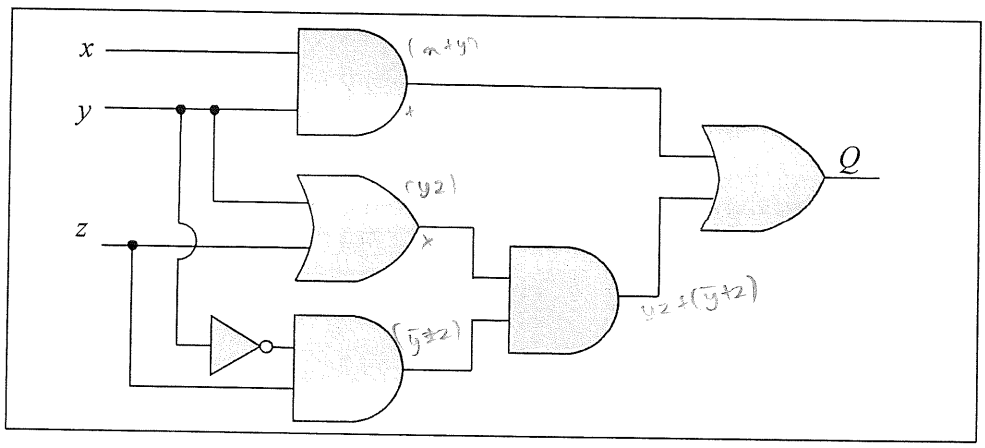


Figure 2

i. Find a Boolean expression that represent the circuit. (2 marks)

ii. Construct a truth table for output Q. (3 marks)

iii. Transfer the output Q to the Karnaugh maps. (2 marks)

iv. Write the minimization of Boolean expression in (b)(iii). (2 marks)

v. Construct a new circuit for (b)(iv). (2 marks)

-- **END OF QUESTIONS** --

FORMULA

Identity	Name
$\overline{\overline{x}} = x$	Law of the double complement
$x + x = x$ $x \cdot x = x$	Idempotent laws
$x + 0 = x$ $x \cdot 1 = x$	Identity laws
$x + 1 = 1$ $x \cdot 0 = 0$	Domination laws
$x + y = y + x$ $x \cdot y = y \cdot x$	Commutative laws
$x + (y + z) = (x + y) + z$ $x \cdot (y \cdot z) = (x \cdot y) \cdot z$	Associative laws
$x + (y \cdot z) = (x + y) \cdot (x + z)$ $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Distributive laws
$\overline{x \cdot y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \cdot \overline{y}$	De Morgan's laws
$x + (x \cdot y) = x$ $x \cdot (x + y) = x$	Absorption laws
$x + \overline{x} = 1$	Unit property
$x \cdot \overline{x} = 0$	Zero property

Complement: $\overline{0} = 1$, $\overline{1} = 0$

Boolean Sum: $1 + 1 = 1$, $1 + 0 = 1$, $0 + 1 = 1$, $0 + 0 = 0$

Boolean Product: $1 \cdot 1 = 1$, $1 \cdot 0 = 0$, $0 \cdot 1 = 0$, $0 \cdot 0 = 0$