CONFIDENTIAL

UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1133
COURSE	: CALCULUS
SEMESTER/SESSION	: 1 – 2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **5** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches whenever required.
4. If in doubt, raise your hand and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

a) Differentiate each of the following function with respect to x.

i. $y = 8x^5 + 5x^2 - \dfrac{2}{e^{2x}}$ (2 marks)

ii. $y = \dfrac{4}{x} + 7\cos x - \ln x$ (2 marks)

iii. $y = (7x-1)^6 + 3$ (2 marks)

iv. $y = x^{10}\ln x$ (use product rule) (3 marks)

v. $y = \dfrac{e^{8x}}{\tan x}$ (use quotient rule) (3 marks)

b) Find $\dfrac{dy}{dx}$ for $4x^3 + xy^3 - y^2 = \sin x$ implicitly. (5 marks)

c) Find the equation of tangent line of $y = e^{2x} + 5$ at the point $x = 0$. (4 marks)

d) Find the stationary point(s) for the curve, $y = x^3 + 3x^2 - 1$ and determine their nature. Hence, sketch the graph. (9 marks)

QUESTION 2

a) Integrate each of the following function.

i. $\int \left(6x^3 + 4e^{2x} - \dfrac{1}{x} \right) dx$ (2 marks)

ii. $\int \left(5\sec^2 x - 14x^{4/3} + 1 \right) dx$ (2 marks)

iii. $\int \dfrac{\ln x}{x}\, dx$ (use by substitution) (4 marks)

iv. $\int x \sin 3x \, dx$ (use by parts) (4 marks)

b) Solve $\int \frac{9x-6}{(x+2)(x^2+4)}dx.$ (10 marks)

Hint: $\int \frac{1}{(x^2+a^2)}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

c) Find the area bounded by the curve $y=4-x^2$ and the line $y=2x+1$. Sketch the graph and label the shaded area. (7 marks)

QUESTION 3

a) Solve $\frac{dy}{dx}-\frac{2x+1}{e^y}=0$ using separation of the variables. (4 marks)

b) Find a general solution of the differential equation, $y'+\frac{y}{x}=e^x$ using the integrating factor method. (8 marks)

QUESTION 4

a) Find a general solution of the following second order homogeneous linear differential equation (ODE).

$$3\frac{d^2y}{dx^2}-7\frac{dy}{dx}=0$$ (3 marks)

b) Consider the following second order linear differential equation (ODE).

$$\frac{d^2y}{dx^2}-4y=e^{2x}+7$$

i. Find the complementary function, y_c. (3 marks)

ii. Find the particular function, y_p. (8 marks)

iii. Find the general solution of the given differential equation. (1 mark)

QUESTION 5

a) Find the inverse Laplace transforms for the following function.

$$L^{-1}\left\{\frac{2s-7}{s^2+16}\right\}$$ (2 marks)

b) Use Laplace transforms to solve the following differential equation.

$$y''+3y'+2y=t, \qquad y(0)=0, \quad y'(0)=1$$ (12 marks)

-- **END OF QUESTION** --

FORMULA

$\frac{d}{dx}(c)=0$	$\int dx = x + C$
$\frac{d}{dx}(x^n)=nx^{n-1}$	$\int k\,f(x)\,dx = k\int f(x)\,dx + C$
$\frac{d}{dx}(kf(x))=k\frac{d}{dx}f(x)$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n\neq -1$
$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}\cdot f'(x)$	$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}+C, \quad n\neq -1$
$\frac{d}{dx}(\sin x)=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x)=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$	$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a}+C$
$\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$	$\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a}+C$
$\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$	$\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a}+C$
$\frac{d}{dx}(e^x)=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}\left(e^{f(x)}\right)=f'(x)e^{f(x)}$	$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b}+C$
$\frac{d}{dx}(\ln x)=\frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(\ln f(x))=\frac{1}{f(x)}f'(x)$	$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}(uv)=uv'+vu'$	$\int u\,dv = uv - \int v\,du$
$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$	$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$	$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)+C$
$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$	$\rho y = \int \rho Q(x)\,dx$ where $\rho = e^{\int P(x)dx}$

TABLE OF PARTICULAR FUNCTION, y_p

Types of $g(x)$	Example of $g(x)$	General solution of y_p
Polynomial	$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$	$A_n x^n + A_{n-1}x^{n-1} + \ldots + A_0$
Exponential	$e^{\alpha x}$	$Ae^{\alpha x}$
Trigonometric	$a\sin\beta x$ $a\cos\beta x$ $a\sin\beta x + a\cos\beta x$	$A\cos\beta x + B\sin\beta x$
Combination of types i,ii,iii	$e^{\alpha x} + a$ $e^{\alpha x} + x^2$ $e^{\alpha x} + a\sin\beta x$	$Ae^{\alpha x} + B$ $Ae^{\alpha x} + Bx^2 + Cx + D$ $Ae^{\alpha x} + B\sin\beta x + C\cos\beta x$

TABLE OF LAPLACE TRANSFORMS

No.	$f(t)$	$L\{f(t)\} = F(s), \quad s > 0$
1.	a	$\frac{a}{s}$
2.	t	$\frac{1}{s^2}$
3.	$t^n,\ n = 2,3,...$	$\frac{n!}{s^{n+1}}$
4.	e^{at}	$\frac{1}{s-a}$
5.	e^{-at}	$\frac{1}{s+a}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cos at$	$\frac{s}{s^2+a^2}$
8.	y'	$sL(y) - y(0)$
9.	y''	$s^2L(y) - sy(0) - y'(0)$