UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1123
COURSE	: ALGEBRA
SEMESTER/SESSION	: 1-2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **9** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

Two complex numbers, z_1 and z_2 are given respectively by $z_1 = 4 - i$ and $z_2 = 2 + 3i$.

a) Express the complex number z where $z = \frac{z_1}{z_2}$ in the form of $a + bi$. (4 marks)

b) Calculate the modulus and argument of z. (4 marks)

c) Express the complex number z in polar form. (1 mark)

QUESTION 2

Find the values of x and y if $(x + iy)(2 + i) = 3 - i$. (5 marks)

QUESTION 3

a) Determine whether $(x+1)$ is a factor of the following polynomials $P(x) = 5x^4 + 2x^3 - 10x^2 + x + 3$. (2 marks)

b) Find the remainder when $P(x) = 8x^3 + 4x + 1$ is divided by $(2x-1)$ using the long division method. Hence, write your answer properly. (5 marks)

c) Express $\frac{x^2 + 5x + 4}{(x+3)(x^2 + 3x + 1)}$ as partial fractions. (8 marks)

QUESTION 4

a) A, B, and L are points in the same horizontal plane, HL is a vertical pole of length 3 meters, AL=5.2m, the angle ALB=113° and the angle of elevation of H from B is 40° in Diagram 1. Calculate the length of LB and AB.

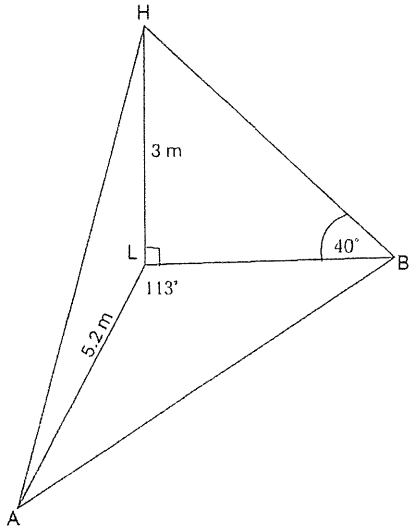

Diagram 1 (6 marks)

b) Solve $2\tan\theta = 1.44$ over the interval $0^\circ \leq \theta \leq 360^\circ$. (3 marks)

c) Show that $3\cos^2\theta = 3 - 3\sin^2\theta$. (2 marks)

QUESTION 5

Diagram 2 shows the journey of a ship that sailed from Port A to Port B and then to Port C. Port B is located 32 km due West of Port A and Port C is 45 km from Port B on a bearing of 40°. Calculate:

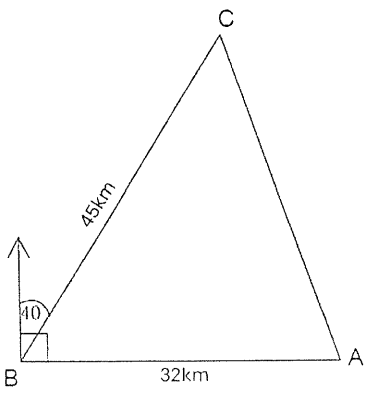

Diagram 2

a) the distance AC. (4 marks)

b) the Angle BAC and ACB. (4 marks)

c) the area of ΔABC. (2 marks)

QUESTION 6

If $A=\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$, $B=\begin{bmatrix} -2 & 3 & 9 \\ 8 & -11 & -34 \\ -5 & 7 & 21 \end{bmatrix}$ and $C=\begin{bmatrix} 3 & 2 \\ 9 & 0 \\ 1 & -1 \end{bmatrix}$. Find:

a) $2A+B$. (3 marks)

b) BC. (4 marks)

c) the inverse of A. (8 marks)

QUESTION 7

Solve the following simultaneous equations using the Gauss Elimination method.

$$2x-3y+z=9$$
$$x+y+z=6$$
$$x-y+z=2$$

(7 marks)

QUESTION 8

The functions f and g are defined as;

$$f(x)=\frac{x+6}{x+2},\ x\in\Re,\ x\neq -2 \text{ and } g(x)=7-2x^2,\ x\in\Re.$$

a) Sketch the graph of $g(x)$ and hence write down its range. (2 marks)

b) Find $(f\circ g)(x)$. (3 marks)

c) Find $f^{-1}(x)$. (4 marks)

d) Solve the equation $f(x)=x$. (5 marks)

QUESTION 9

a) Given $\vec{p} = -2i + 4j + 7k$ and $\vec{q} = -i + 3j + 8k$. Find the angle between $\left(\vec{p} + \vec{q}\right)$ and $\left(\vec{p} - \vec{q}\right)$. (7 marks)

b) If $\left|\vec{a}\right| = \sqrt{34}$, $\left|\vec{b}\right| = \sqrt{35}$ and the angle between $\vec{a}$ and $\vec{b}$ is 66°. Find the value of $\vec{a} \bullet \vec{b}$. (2 marks)

c) Given two vectors, $\vec{u} = -7i + j$ and $\vec{v} = 7i - 5j - k$. Find the area of the parallelogram formed by $\vec{u}$ and $\vec{v}$. (5 marks)

----------END OF QUESTIONS----------

FORMULA	
$z = a + bi, \quad \bar{z} = a - bi$	$z = r(\cos\theta + i\sin\theta)$
$r = \lvert z\rvert = \lvert a + b_i\rvert = \sqrt{a^2 + b^2}$	$\theta = Arg(x) = \tan^{-1}\left\lvert\frac{b}{a}\right\rvert$
1^{st} Quadrant: $\theta = \tan^{-1}\left\lvert\frac{b}{a}\right\rvert$ 2^{nd} Quadrant: $\theta = \pi - \tan^{-1}\left\lvert\frac{b}{a}\right\rvert$	3^{rd} Quadrant: $\theta = -\left[\pi - \tan^{-1}\left\lvert\frac{b}{a}\right\rvert\right]$ 4^{th} Quadrant: $\theta = -\tan^{-1}\left\lvert\frac{b}{a}\right\rvert$
$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n$ $= r^n(\cos n\theta + i\sin n\theta)$	$z = re^{i\theta}$
$f(x) = d(x)q(x) + r(x)$	$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
$a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$	$Area = \frac{1}{2}ab\sin C$ $= \frac{1}{2}ac\sin B$ $= \frac{1}{2}bc\sin A$
$A^{-1} = \frac{1}{\lvert A\rvert}adj(A)$	$u \times v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} i - \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} j + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} k$ $= (b_1c_2 - b_2c_1)i - (a_1c_2 - a_2c_1)j + (a_1b_2 - a_2b_1)k$
$\lvert\vec{u}\rvert = \sqrt{a^2 + b^2 + c^2}$	$\hat{u} = \frac{u}{\lvert u\rvert} = \frac{ai + bj + ck}{\sqrt{a^2 + b^2 + c^2}}$
$\theta = \cos^{-1}\left[\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right]$	$u \cdot v = a_1a_2 + b_1b_2 + c_1c_2$
$\lvert\vec{u}\times\vec{v}\rvert = \lvert\vec{u}\rvert\lvert\vec{v}\rvert\sin\theta$	$\cos(-\theta) = \cos(\theta)$
$\tan(-\theta) = -\tan(\theta)$	$\sin(-\theta) = -\sin(\theta)$